

Q1:

Mid term Solution (Antenna)

$$1) \quad I = I_0 \left[e^{-\frac{z^2}{L}} + \cos \frac{\pi z}{L} \right]$$

$$A = \frac{\mu}{4\pi} \iiint_V J(x, y, z) \frac{-e^{-jkR}}{R} dv$$

$$A = \frac{\mu I}{4\pi} \int_I dz \frac{-e^{-jk\hat{r}}}{r} = \frac{\mu I_0}{4\pi r} e^{-jkcr} \left[\frac{-\frac{2}{L}z^2}{-2jL} + \sin \frac{\pi z}{\pi/L} \right] \Big|_0^L$$

$$A_z = \frac{\mu I_0 e^{-jkcr}}{4\pi r} \left[\frac{-1}{\frac{-2}{L}L} - \frac{1}{\frac{-2}{L}L} + \frac{2L}{\pi} \right]$$

$$A_z = \frac{\mu I_0 e^{-jkcr}}{4\pi r} \Big|_I$$

$$A_r = A_z \cos \theta = \frac{\mu I_0 I e^{-jkcr}}{4\pi r} \cos \theta$$

$$A_\theta = A_z \sin \theta (-\sin \theta) = -\frac{\mu I_0 I e^{-jkcr}}{4\pi r} \sin \theta$$

$$2) \quad E_\theta = j \gamma k \frac{I_0 L}{4\pi r} e^{-jkcr} \sin \theta$$

$$\omega_{rad} = \frac{|E|^2}{2\gamma} = \frac{2k^2 I_0^2 L^2}{32 \pi^2 r^2} \sin^2 \theta$$

$$U = r^2 \omega_{rad} = \frac{2k^2 I_0^2 L^2 \sin^2 \theta}{32 \pi^2}$$

$$P_{rad} = \iint_0^{2\pi} \int_0^\pi U \sin \theta d\theta d\phi$$

$$= 2\pi U_{max} \int_0^\pi \sin^3 \theta d\theta = \frac{2\pi U_{max} (4)}{3}$$

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$$= \frac{2\pi^2 k^2 I_0^2 L^2}{32 * 3\pi^2} = \frac{2k^2 I_0^2 L^2}{12\pi}$$

$$P_{rad} = \frac{1}{2} I_0^2 R_r \quad \therefore R_r = \frac{2P_{rad}}{I_0^2}$$

$$R_r = \frac{2\gamma k^2 I_0^2 L^2}{12\pi I_0^2} = \frac{\gamma k^2 L^2}{6\pi} = 20 k^2 L^2$$

$$= 20 \left(\frac{2\pi}{\lambda}\right)^2 L^2$$

$$\rightarrow R_r = 8_0 \pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$\rightarrow D = 4 \frac{\pi U_{max}}{P_{rad}} = \frac{4\pi \gamma k^2 I_0^2 L^2 + 112\pi}{32\pi^2 r^2 + 2k^2 I_0^2 L^2}$$

$$= 1.5 \quad \text{not}$$

$$(3) \quad L = \frac{\lambda}{20}, R_L = 5\Omega, X_A = 3\Omega$$

$$T.L \rightarrow L = \frac{\lambda}{4}, Z_C = 50\Omega$$

$$V_g = 1V, R_g = 40, X_g = 50\Omega$$

lossless T.L, $R_L = 0$

$$Z_{in} = Z_C \frac{Z_L + jZ_C \tan \beta L}{Z_C + jZ_L \tan \beta L}$$

$$\beta L = \frac{2\pi}{\lambda} * \frac{\lambda}{4} = \frac{\pi}{2} \rightarrow \tan \left(\frac{\pi}{2}\right) = \infty$$

$$Z_{in} = \frac{Z_C^2}{Z_L} = (50)^2 / 50 = \frac{50^2}{5+j3}$$

$$= 367.7 - j220.59 \Omega$$

(2)

$$|I| = \frac{1}{\sqrt{(367.7 + 40)^2 + \sqrt{(10 + 250 \cdot 5g)^2}}} \quad \text{Circuit diagram: } R_g \parallel \left(x_g \parallel \left(s \parallel x_{in} \right) \right) \parallel R_{in}$$

$$= 2.11 \text{ A}$$

$$P_r = e_0 * P_{in} = 1/2 |I|^2 R_{in} = 0.5 (2.11)^2 (367.7) = 818.5 \text{ W}$$

$$P_g = \frac{1}{2} |I|^2 R_g = 0.5 (2.11)^2 \cdot 40 = 89.642 \text{ W}$$

$$P_T = P_r + P_g = 907.5 \text{ W}$$

$$\text{b) } P_r = P_{in} = 0.5 (2.11)^2 (367.7) = 818.5 \text{ W}$$

$$\text{c) } G = eD = \frac{4\pi}{P_{rad}} \text{ max } \rightarrow P_{rad} = \int \int S \cdot r^2 \Theta d\Theta d\phi$$

$$= \pi \int_0^\pi (1 - \cos 2\theta) d\theta = \pi^2$$

$$G = \frac{4\pi}{\pi^2} = \frac{4}{\pi}$$

$$A = G \frac{\lambda^2}{4\pi} = \frac{4}{\pi} \cdot \frac{\lambda^2}{4\pi} = \frac{\lambda^2}{\pi}$$

$$\textcircled{1} \quad \text{HPBW} \rightarrow \sin \Theta = \frac{1}{2}, \Theta_h = 30^\circ$$

$$\text{HPBW} = 2\Theta_h = 6^\circ$$

$$F_N B_w \rightarrow \sin G = 0, \Theta_n = 0$$

$$F_N B_w = 0, SLL = \frac{D_{max}}{D_{SLL}} = \frac{4\pi}{0} = \infty$$

$$\text{Ans: Farfield distance} = \frac{2d^2}{\lambda} = \frac{2L^2}{\lambda}$$

$$= 2 \left(\frac{\pi^2}{(\lambda/2)^2} \right) = \frac{2\pi^2}{\lambda/2}$$

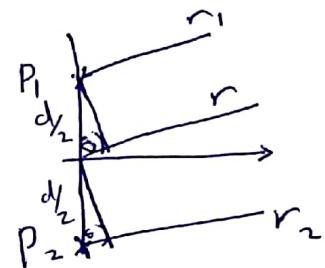
at $L = \lambda/2$.

$\Phi_2:$

$$E_{\text{total}} = E_1 + E_2$$

$$= \frac{-e^{jkr_1}}{r_1} + \frac{-e^{jkr_2}}{r_2}$$

$$\text{at } R_1 = R_2 = R = r$$



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$$r_1 = r - d/2 \cos \theta$$

$$r_2 = r + d/2 \cos \theta$$

$$E_{\text{total}} = \frac{e^{-jk(r+d/2 \cos \theta)}}{r} + \frac{e^{-jk(r-d/2 \cos \theta)}}{r}$$

$$= \frac{-e^{-jkr}}{r} \left[e^{\frac{-jkd/2 \cos \theta}{r}} + e^{\frac{jkd/2 \cos \theta}{r}} \right]$$

E_P AF

$$AF_n = \frac{2 \cos \left(\frac{kd}{2} \cos \theta \right)}{\lambda} = \cos \left(\frac{kd}{2} \cos \theta \right)$$

$$\rightarrow \text{minimum } \cos \left(\frac{kd}{2} \cos \theta \right) = 0 \Rightarrow \theta_n = 0^\circ, k = \frac{2\pi}{\lambda}, d = \frac{\lambda}{2}$$

$$\rightarrow \text{maximum } \cos \left(\frac{kd}{2} \cos \theta \right) = \pm 1 \rightarrow \theta_{\max} = 90^\circ$$

$$\rightarrow \theta_n \rightarrow \cos \left(\frac{kd}{2} \cos \theta \right) = \frac{1}{\sqrt{2}}, \theta_n = 60^\circ$$

$$\text{HPBW} = 60^\circ, \text{ FNBW} = 180^\circ$$

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